# Spherical Centroidal Voronoi Tessellations: Theory, Analysis and Practical Issues Part I

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# Outline

- Motivation
- Examples to Support Motivation
- A Framework: SCVT
  - Definition and Examples
  - Theoretical Basis
- SCVT for climate system modeling
- SCVT testbed
- Conclusions

# Motivation

The climate modeling community is becoming increasingly interested in capturing the regional character of global climate change. Examples will follow.

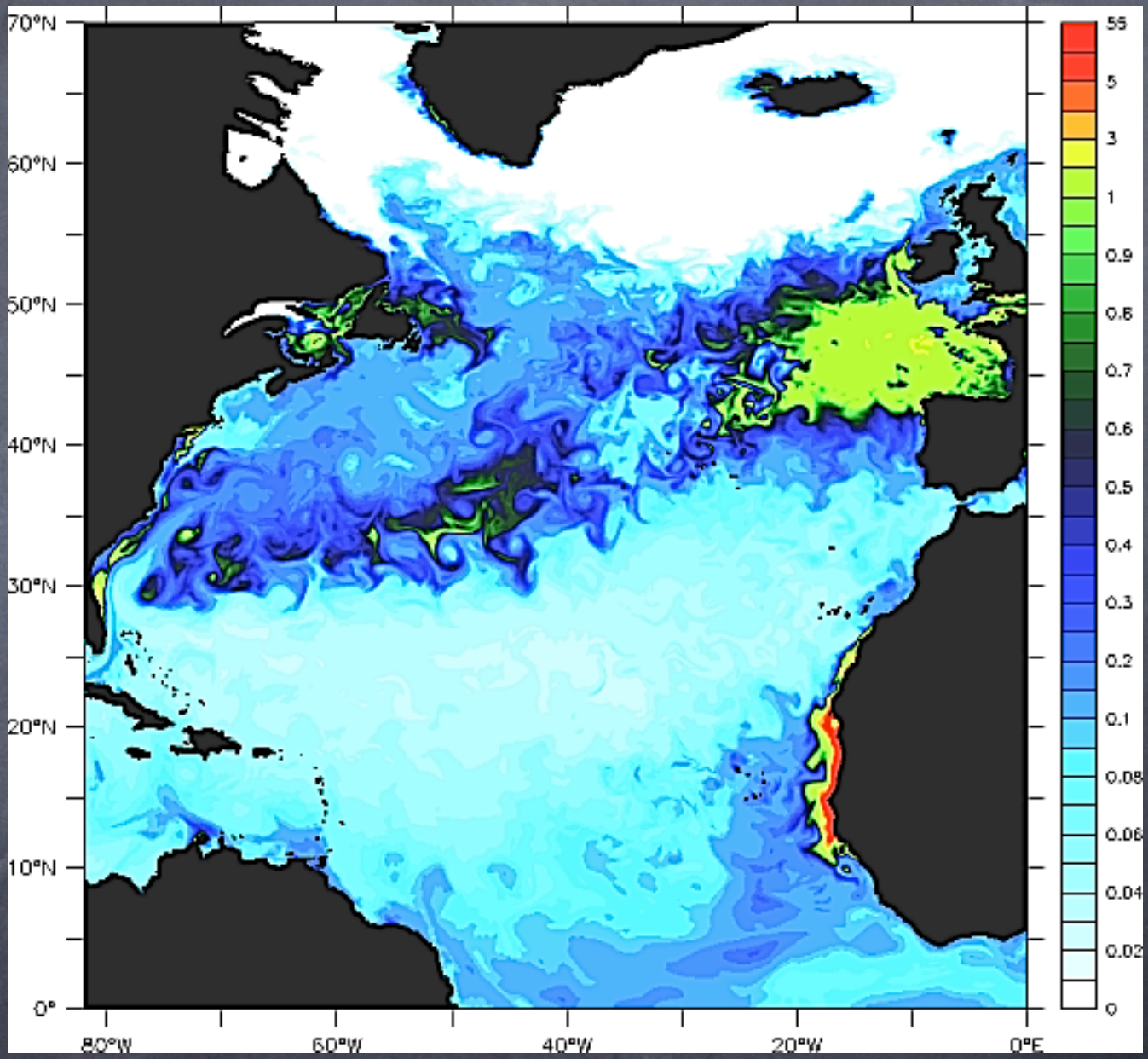
This interest is somewhat orthogonal to the traditional approach of quasi-uniform, global climate modeling.

Can we accommodate both high-resolution regional processes and traditional global modeling within a single framework?

# Ocean Eddies: Biogeochemistry

Ecological processes are becoming a primary motivation for global ocean modeling as we attempt to understand the impact of climate change on the world's oceans.

These processes are highly coupled and strongly nonlinear. Accurate simulation my require resolving ocean eddy processes.



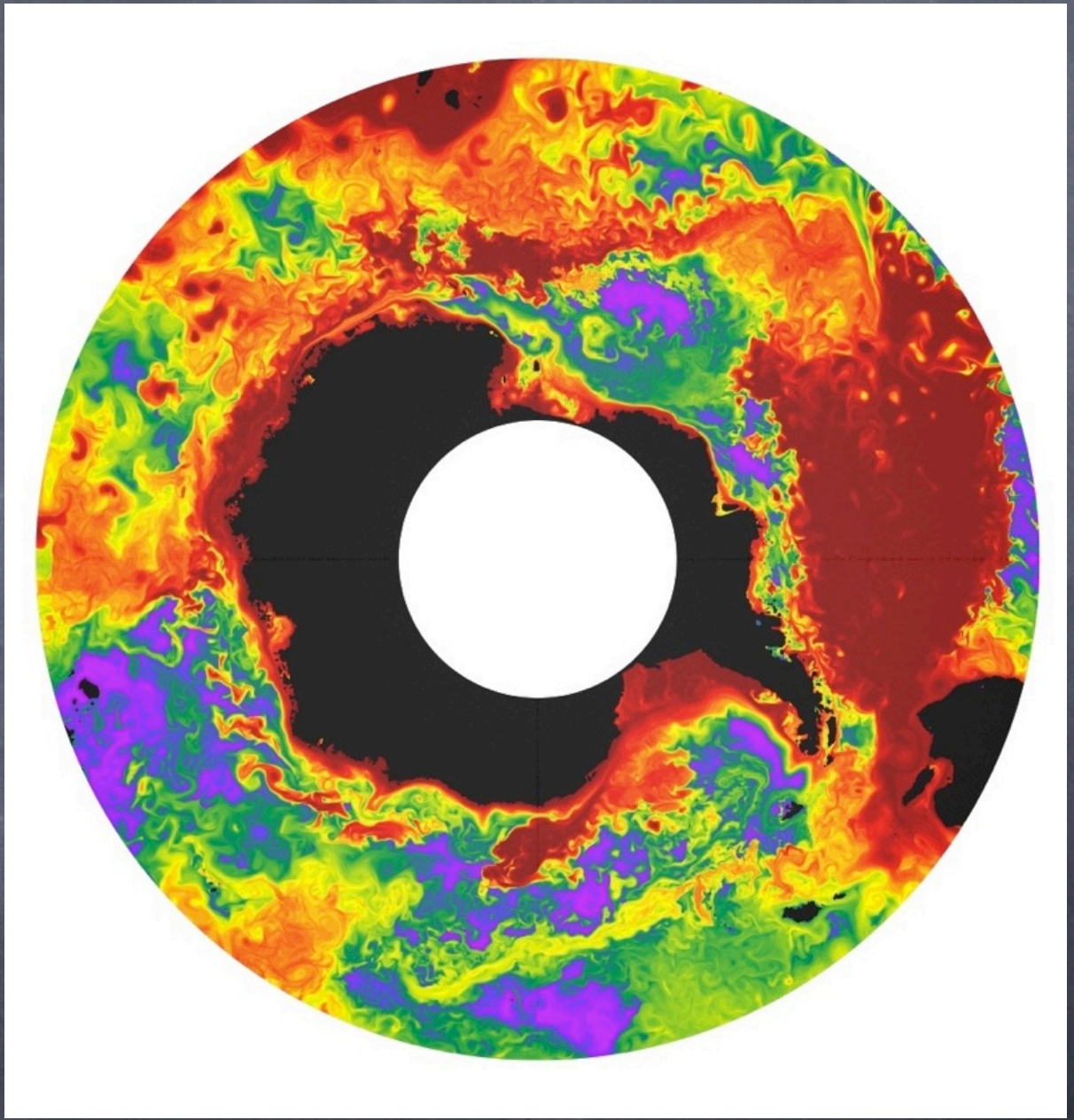

Diatom concentration in 1/10 deg simulation (from Maltrud)

# Ocean Eddy -Atmosphere Interaction

Changes in mean wind stress (driven by climate change) lead to significant changes in the ocean circulation and water mass distribution.

The changes in ocean circulation are primarily eddy driven.

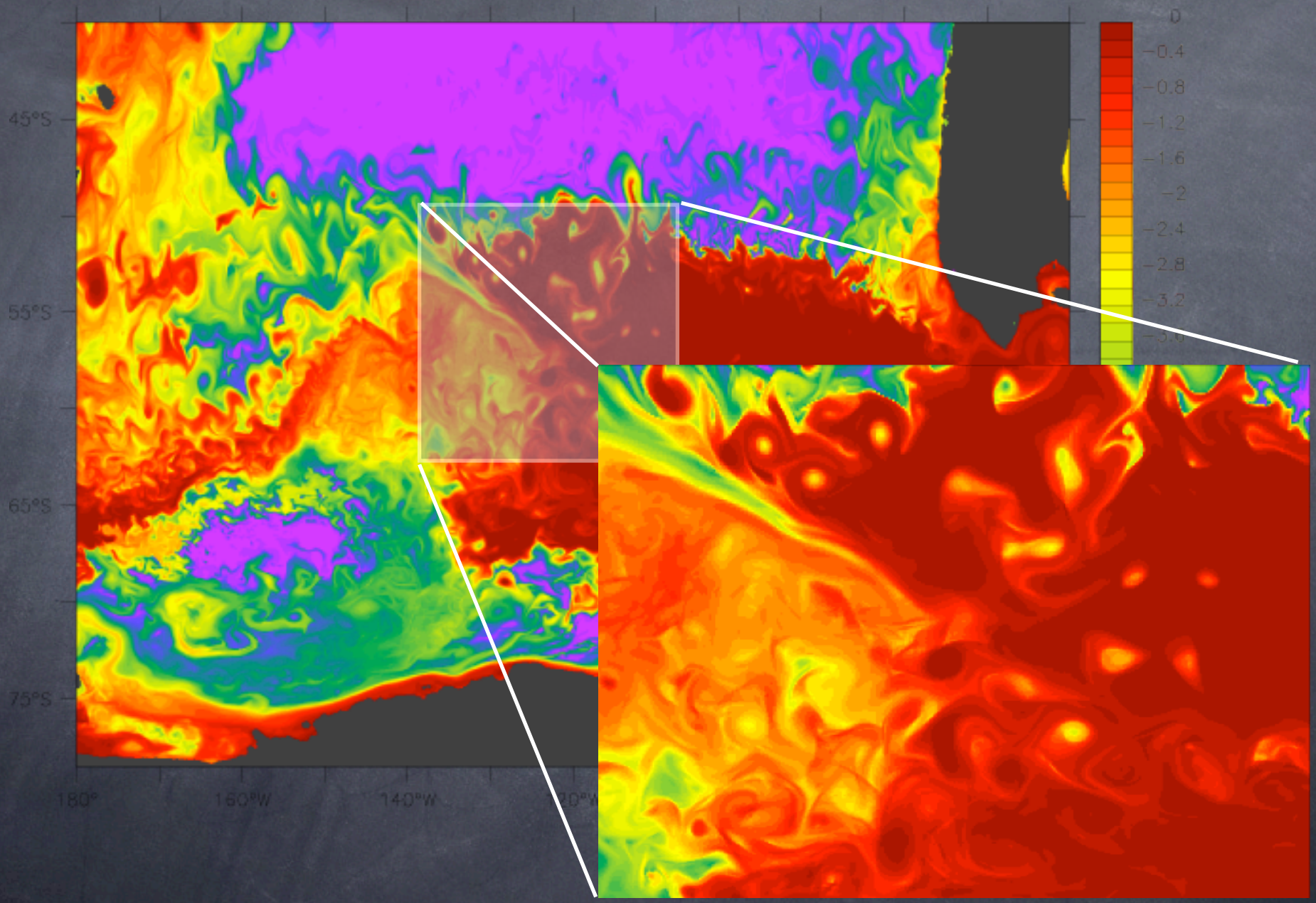
Turbulence closure schemes are presently not able to robustly model these eddy-driven changes to the mean ocean structure.



Passive tracer in global 1/10 deg simulation (Maltrud)

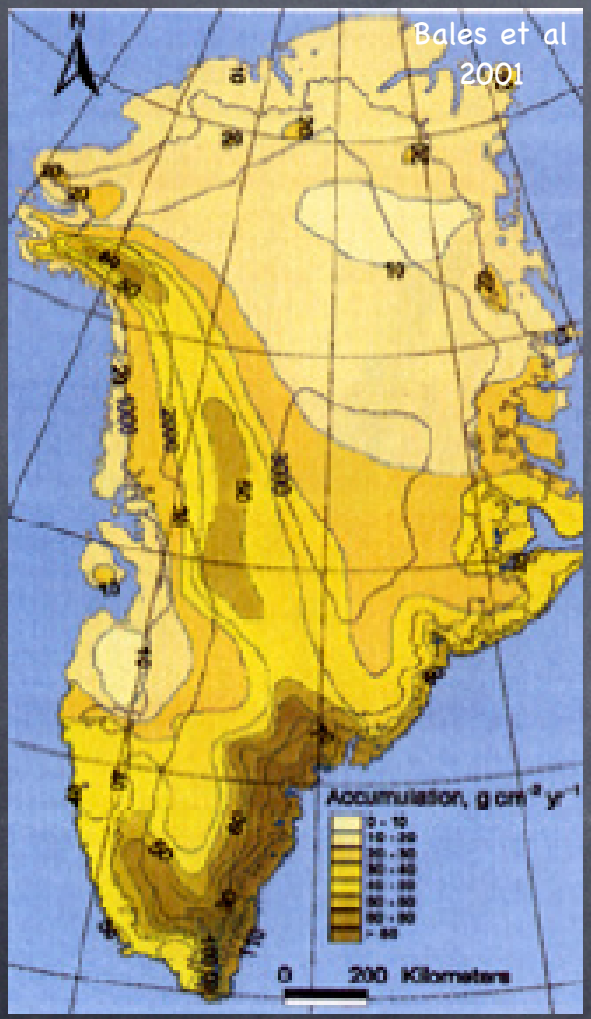
# Ocean Eddy-Atmosphere Interaction

## The closer we look, the more structure we find.



# Ice Sheets and Sea Level Rise

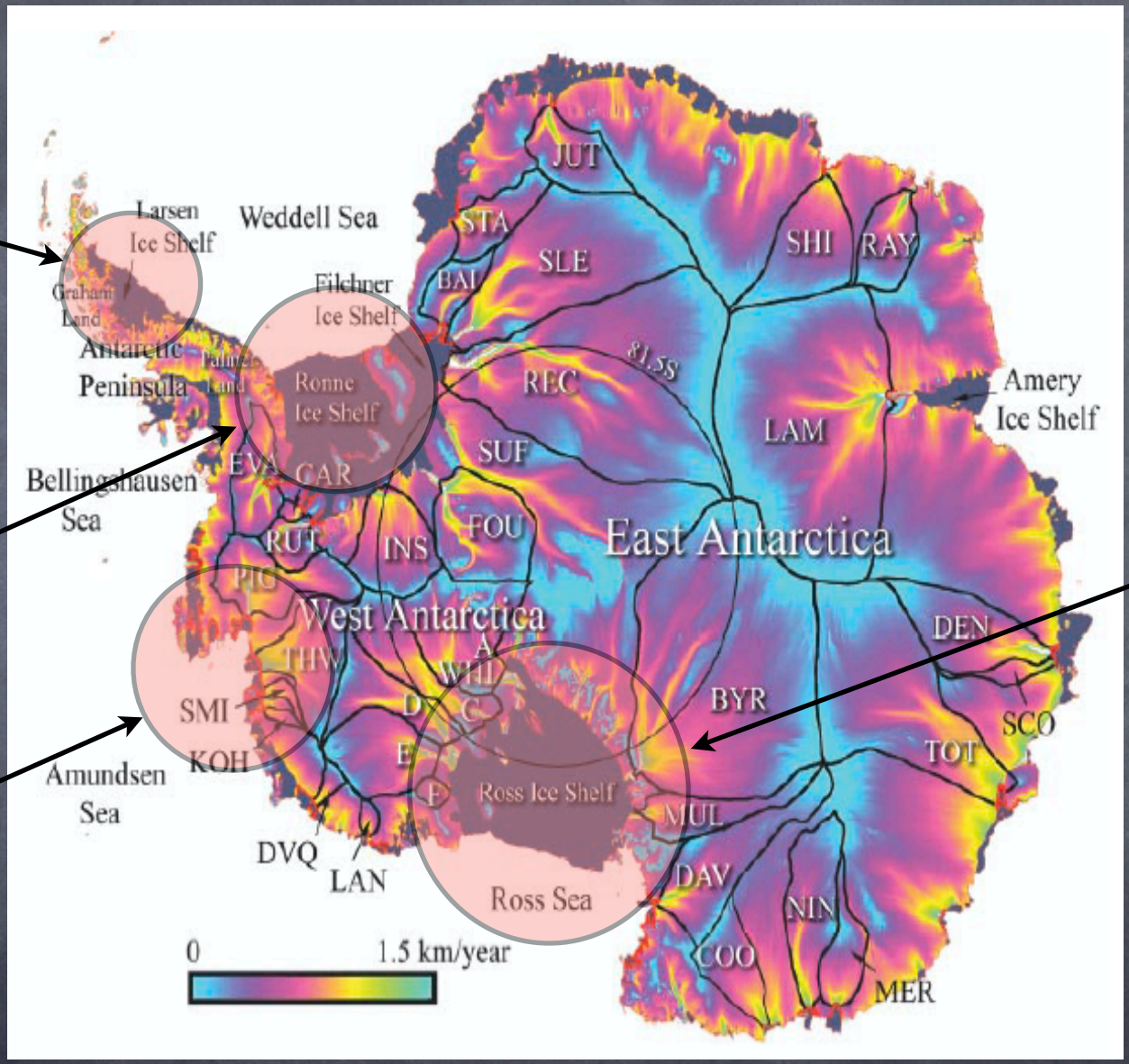
Greenland holds about 7 m of sea-level rise.




Yet extremely small scales may be critical to the evolution of the ice sheet.

We need to understand sea level rise, but some of the processes that control sea level rise occur at scales far finer than are resolved in traditional climate models.

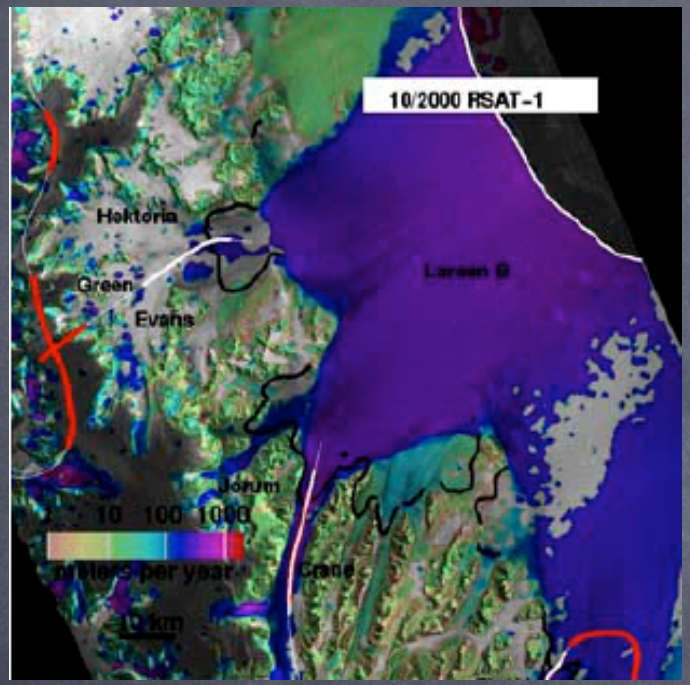
# Ice Shelf-Ocean Interaction

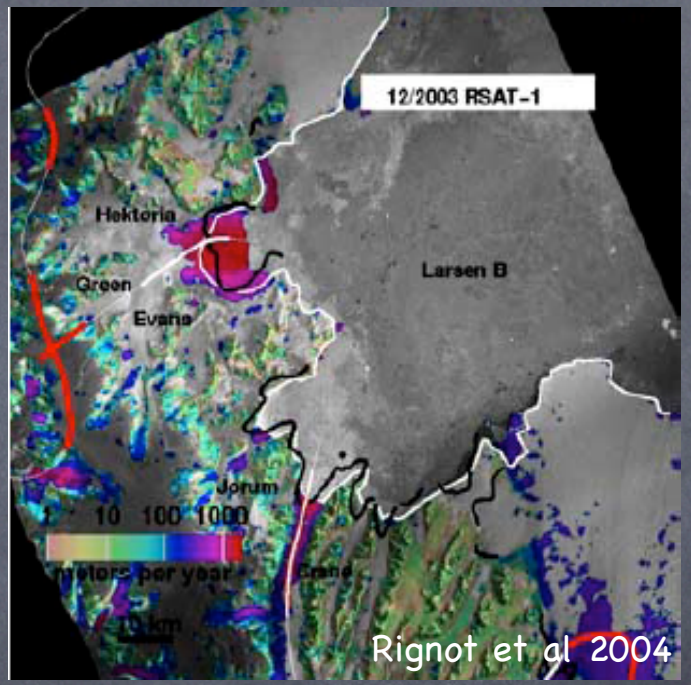



Ice shelves provide buttressing to grounded ice sheets that slows the rate of ice flow into the ocean.

# The Larsen B ice shelf provides one example.

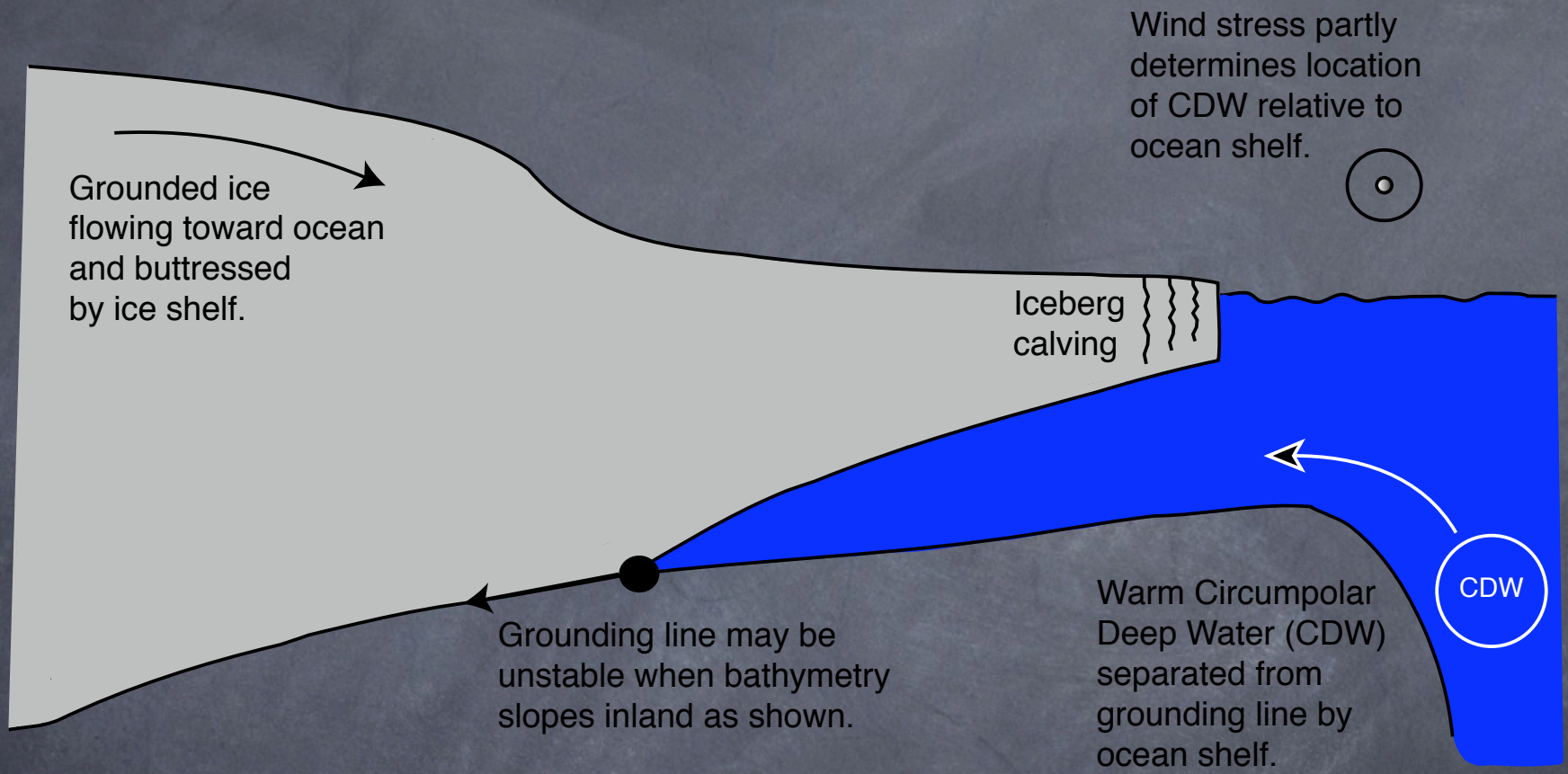
The glaciers buttressed by Larsen B sped up
by a factor of 8 in 2002 following its collapse.




What scales and processes are required to accurately capture these ice shelf-ocean interactions?

# Critical processes in ice shelf-ocean interaction




Typical scales here are order km -- meaning resolutions down to even 1 km might be required.

So, in summary, there are a set of processes that

1) are likely critical for understanding global climate change
2) that can not be accommodated in traditional modeling approaches.

- Ocean eddies for biogeochemistry/ecology
- Ocean eddies for ocean/atmos interaction
- Ice streams for basal sliding of grounded ice sheets
- Ice shelf-ocean interaction for grounded ice flow

How can we capture these processes, yet still retain a computationally-tractable modeling system?

# Our approach is to identify a single framework that has the potential to be applicable to a wide class of problems in climate science.

Some properties we are looking for in this modeling framework:

- Robust at long time scales (i.e. conservative).
- Robust meshes for variable resolution and/or adaptive modeling.
- Prior knowledge of mesh properties and mesh quality.
- Flexible enough to be applicable to a wide class of systems.

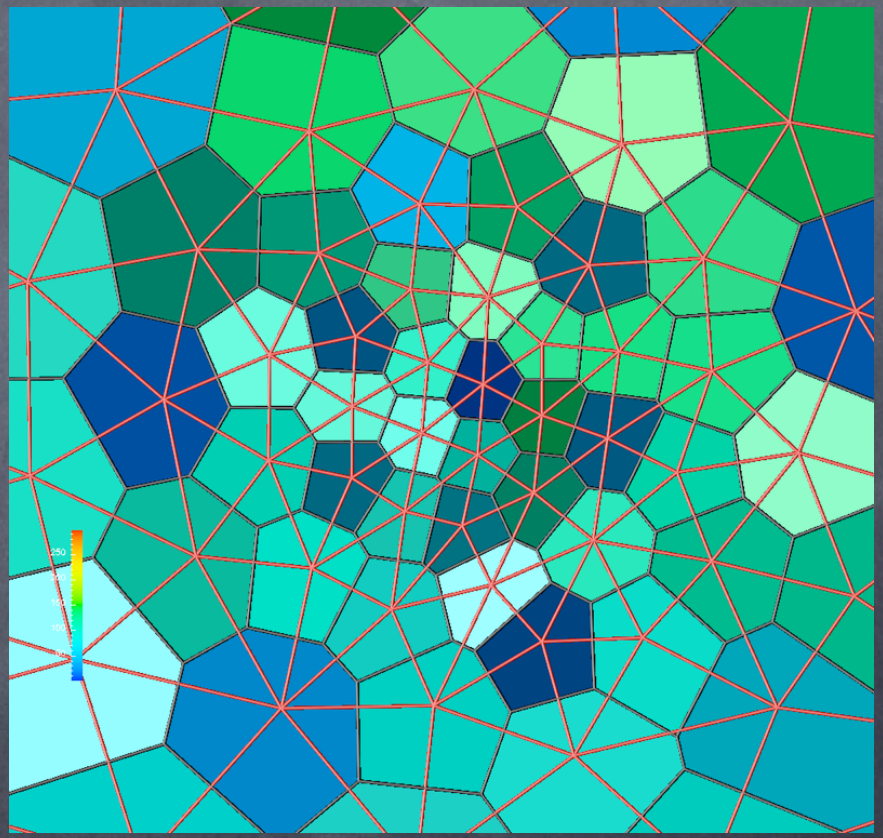
Of course there are several potentially productive routes to meet our goals -- and here we explore just one of these paths.

Here we will explore the use of Spherical Centroidal Voronoi Tessellations (SCVT) for application to climate system models where variable and/or adaptive resolution is required.

SCVTs and their close relatives have already been successfully used in climate system modeling.

To date, the primary motivation for their use has been the mesh uniformity when tiling the sphere.

I think their potential goes well beyond the ability to produce a globally uniform mesh.



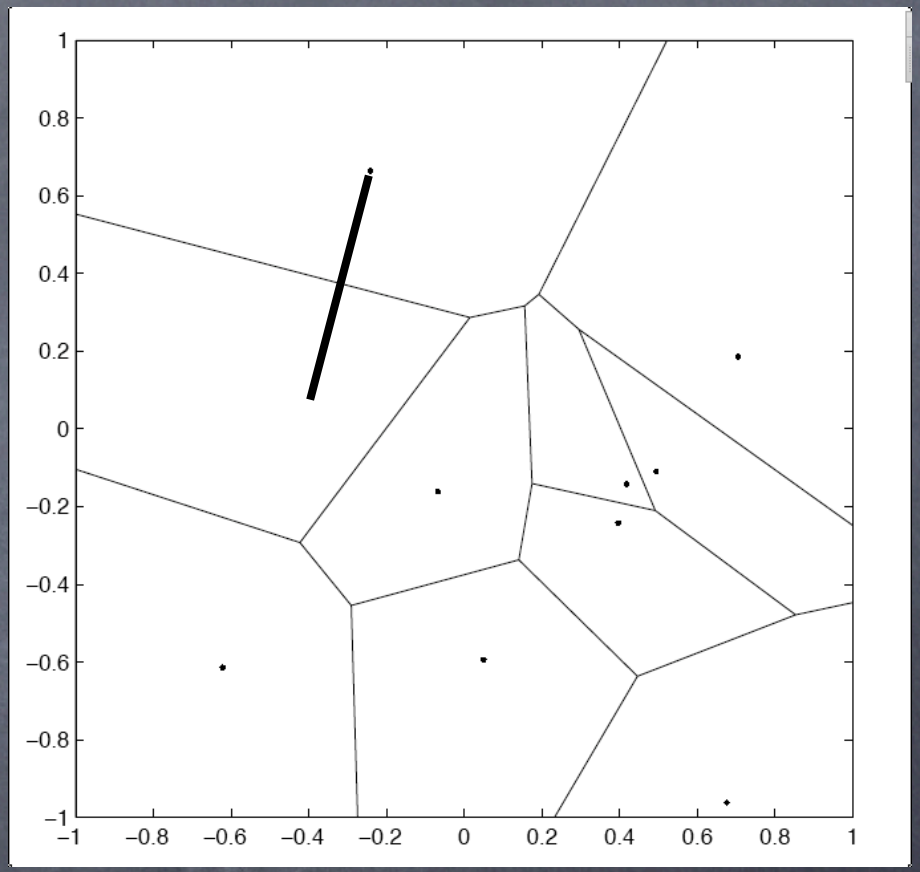
Voronoi Diagram (colored)
and the dual Delaunay triangulation

# Definition of a Voronoi Tessellations

Given a region, S
And a set of generators, $z_i$ ...

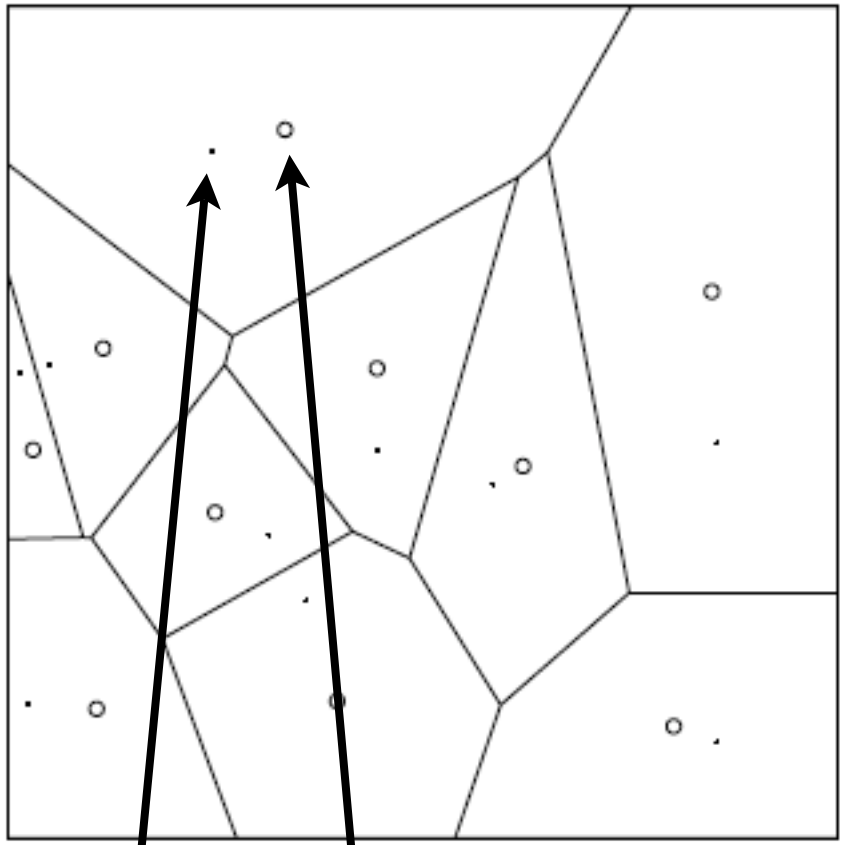
The Voronoi region, $V_i$, for each $z_i$ is the set of all points closer to $z_i$ than $z_j$ for j not equal to i.

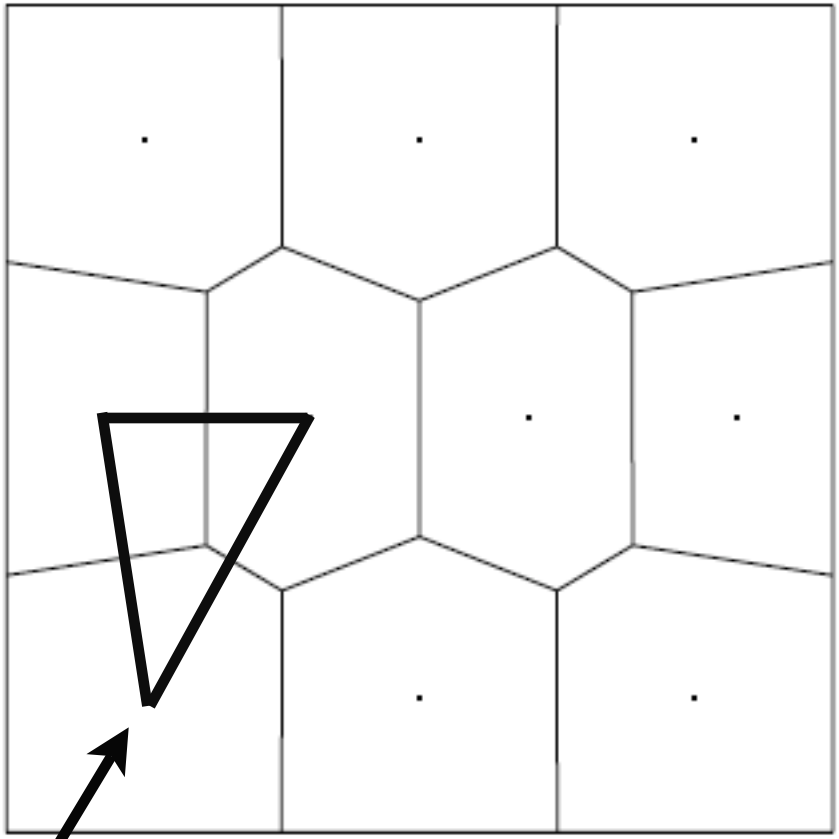
We are guaranteed that the line connecting generators is orthogonal to the shared edge and is bisected by that edge.




But this does not mean that the grid is nice ....

# Definition of a Centroidal Voronoi Tessellations





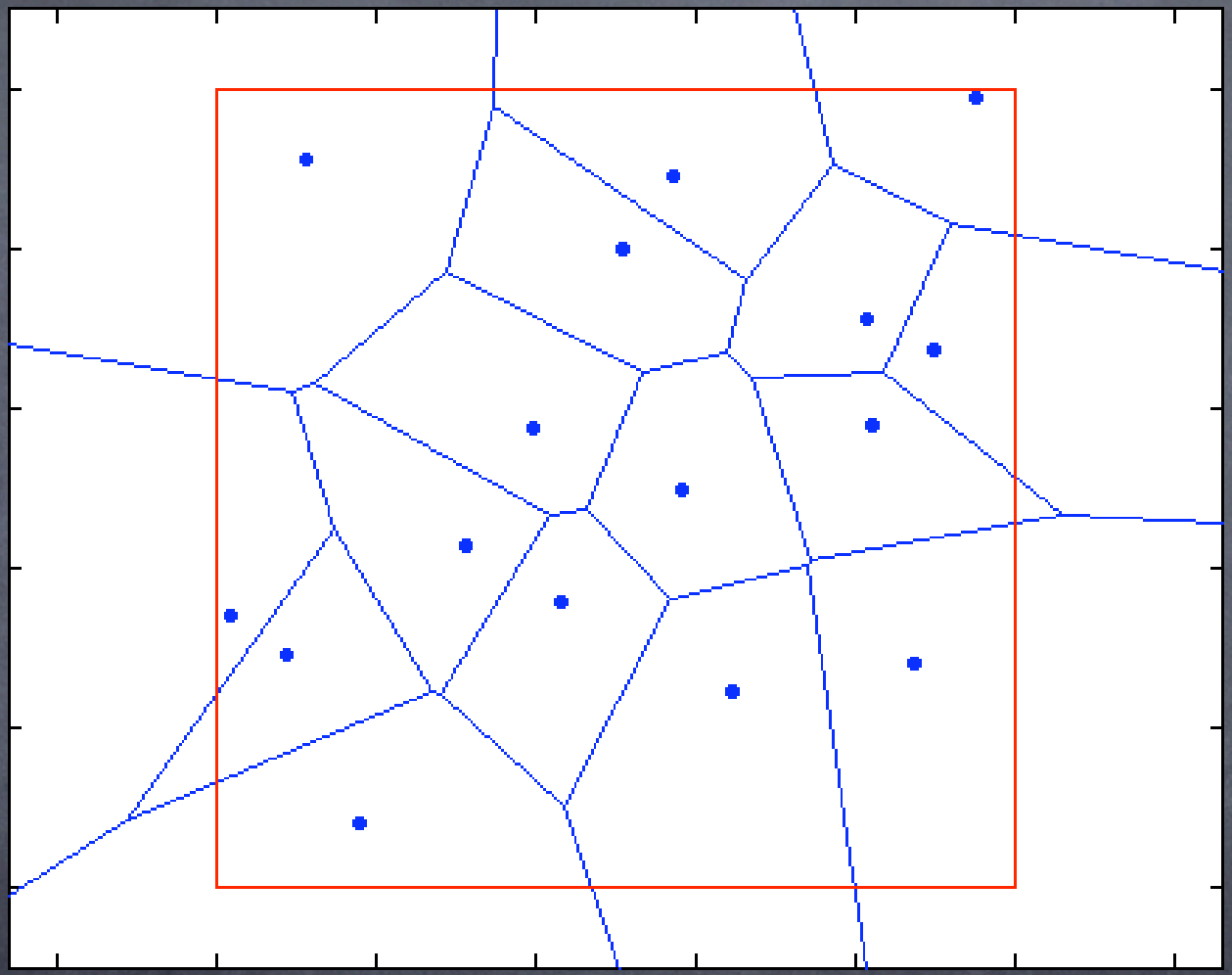
$z_i$

$z_i^*$ = center of mass wrt a user-defined density function

Dual tessellation

$$z^* = \frac{\int_V w\rho(w)\,dw}{\int_V \rho(w)\,dw}$$

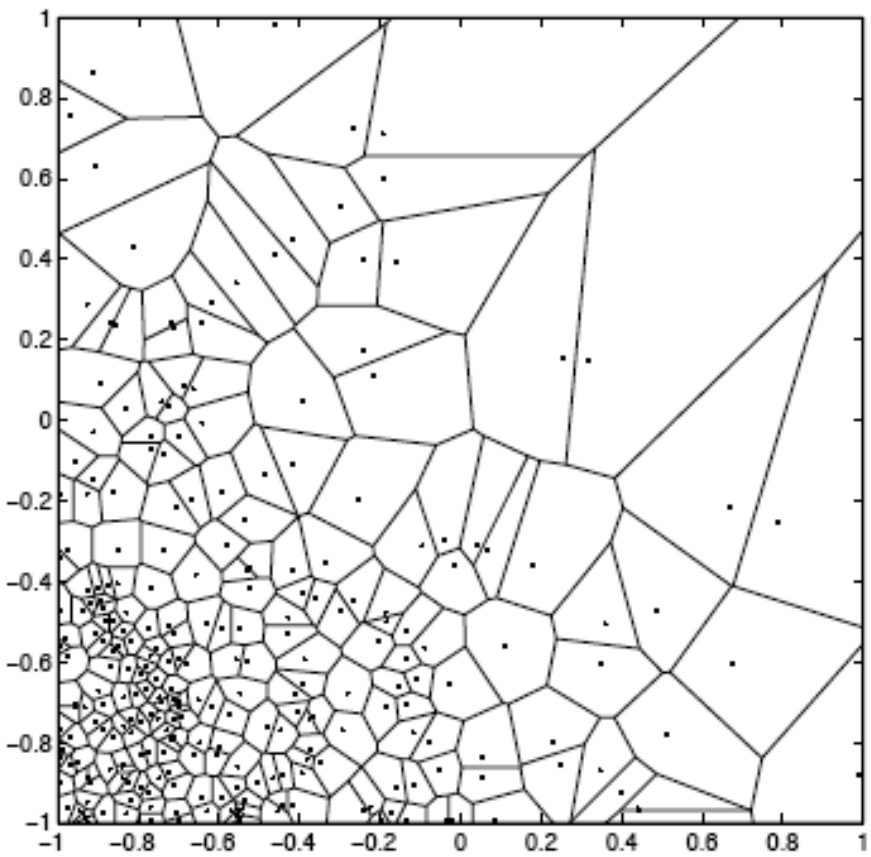
# Iterating toward and CVT ....



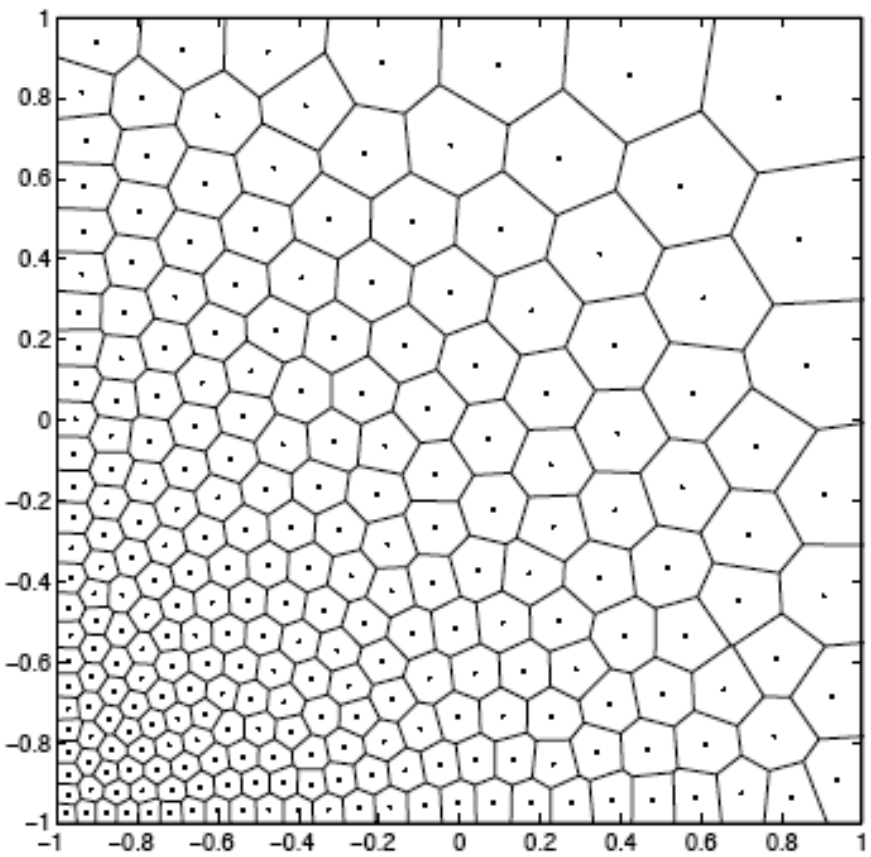
# Non-uniform Centroidal Voronoi Tessellations

Distribute generators in such a way as to make the grid regular.

Also biases the location of those generators to regions of high density.




Random sampling




Centroidal Voronoi

Critical Point: The user gets to pick the density function!

# (S)CVTs have their roots in applied math …

Gersho conjecture (now proven in 2D): as we added generators, all cells evolve toward perfect hexagons. Meaning that the grid just keeps getting more regular as we add resolution.

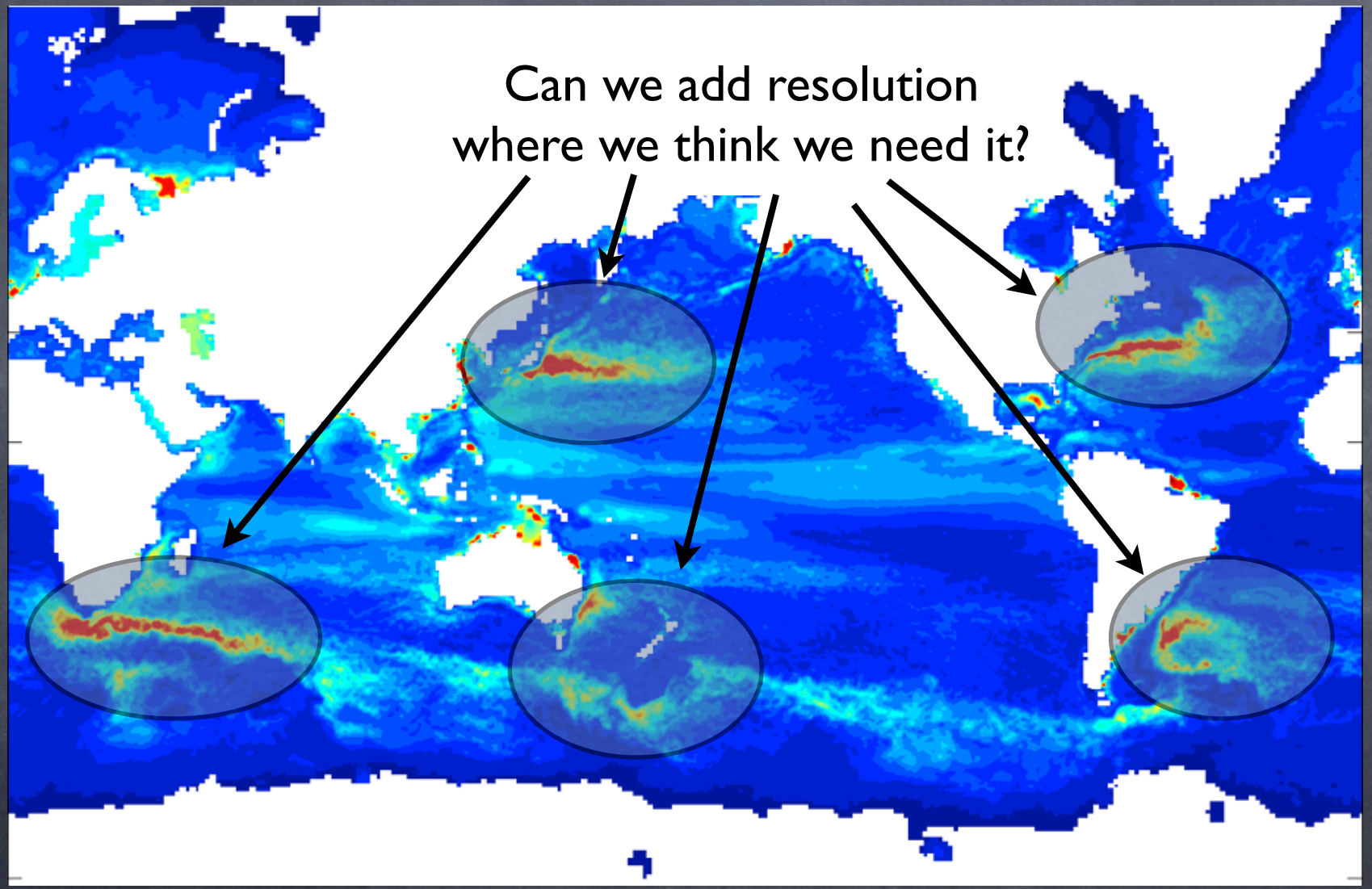
Optimal sampling: given a region, R, and N buckets to measure precipitation in R, the optimal placement of those buckets is a CVT. If a prior distribution, P, of precipitation is known, the CVT takes that information into account with rho=sqrt(P).

Guaranteed to have 2nd-order truncation error of Poisson equation.

In summary: if Voronoi tessellations are to be used, then there is no good reason not to use Centroidal Voronoi Tessellations.

So what do these grids look like in practice?

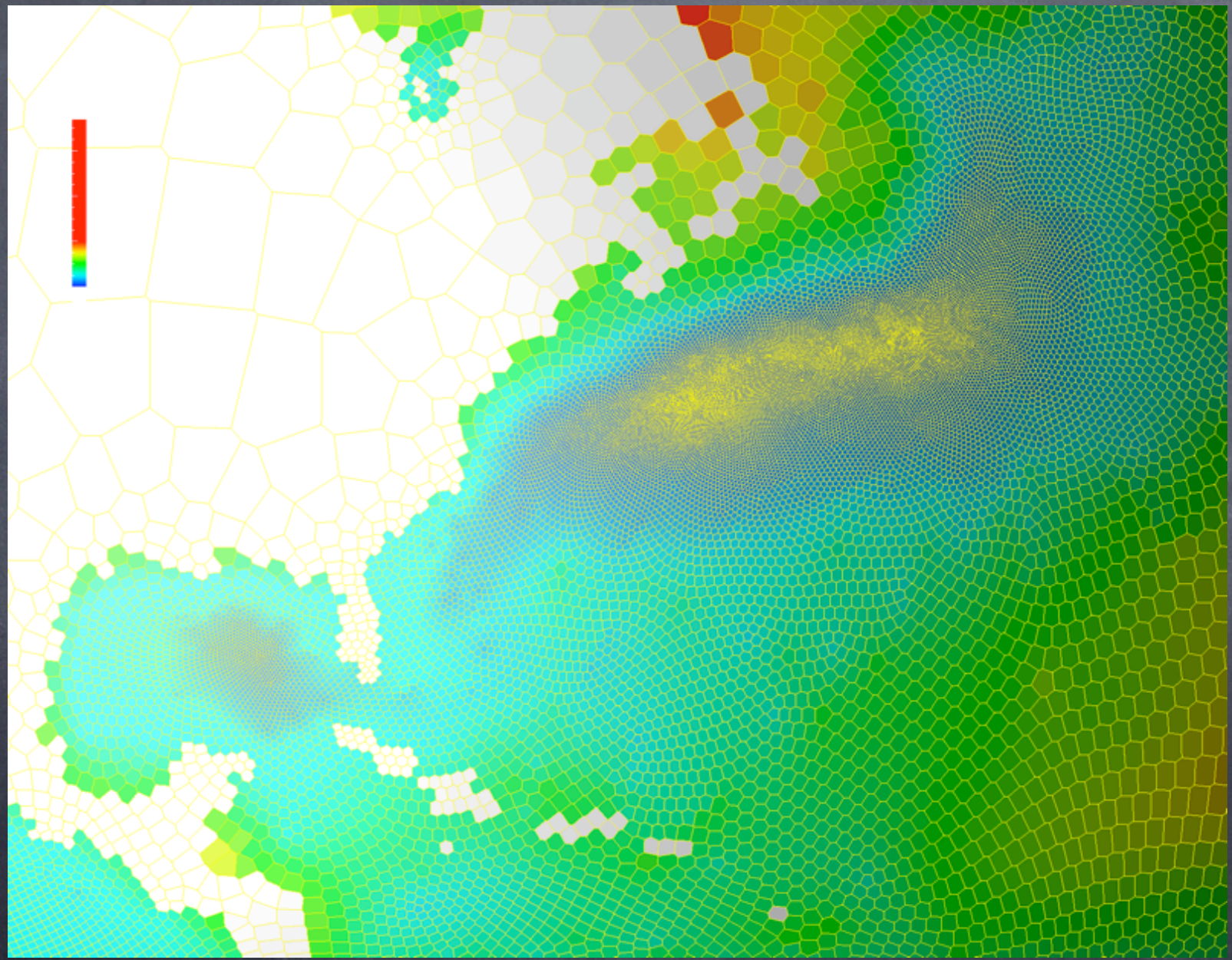
# Resolving regions of strong eddy activity.



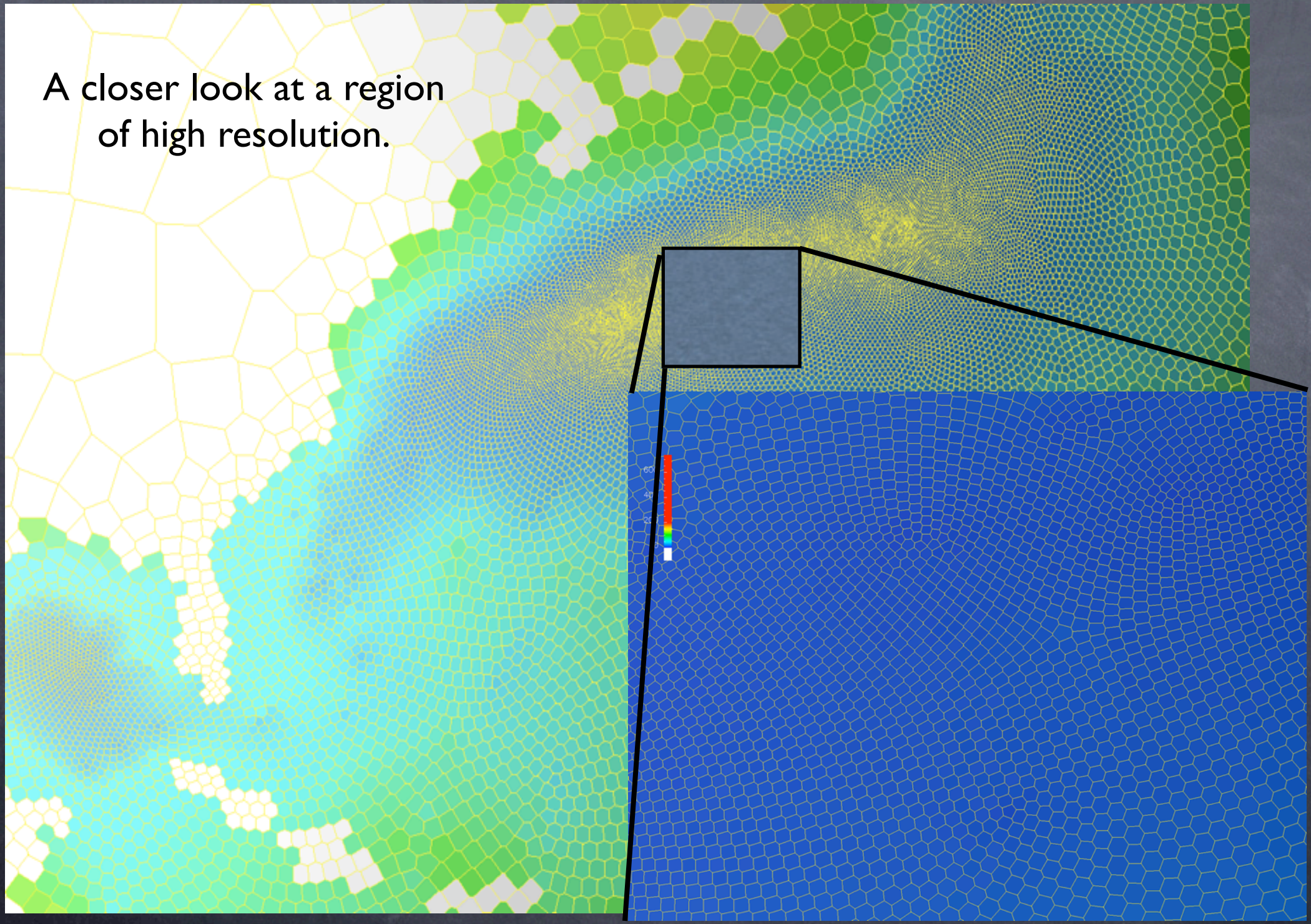

Time-mean sea-surface height variability is a proxy for eddy activity.

# An example of an SCVT

## the density proxy use here was TOPEX SSH Variance



A closer look at a region of high resolution.



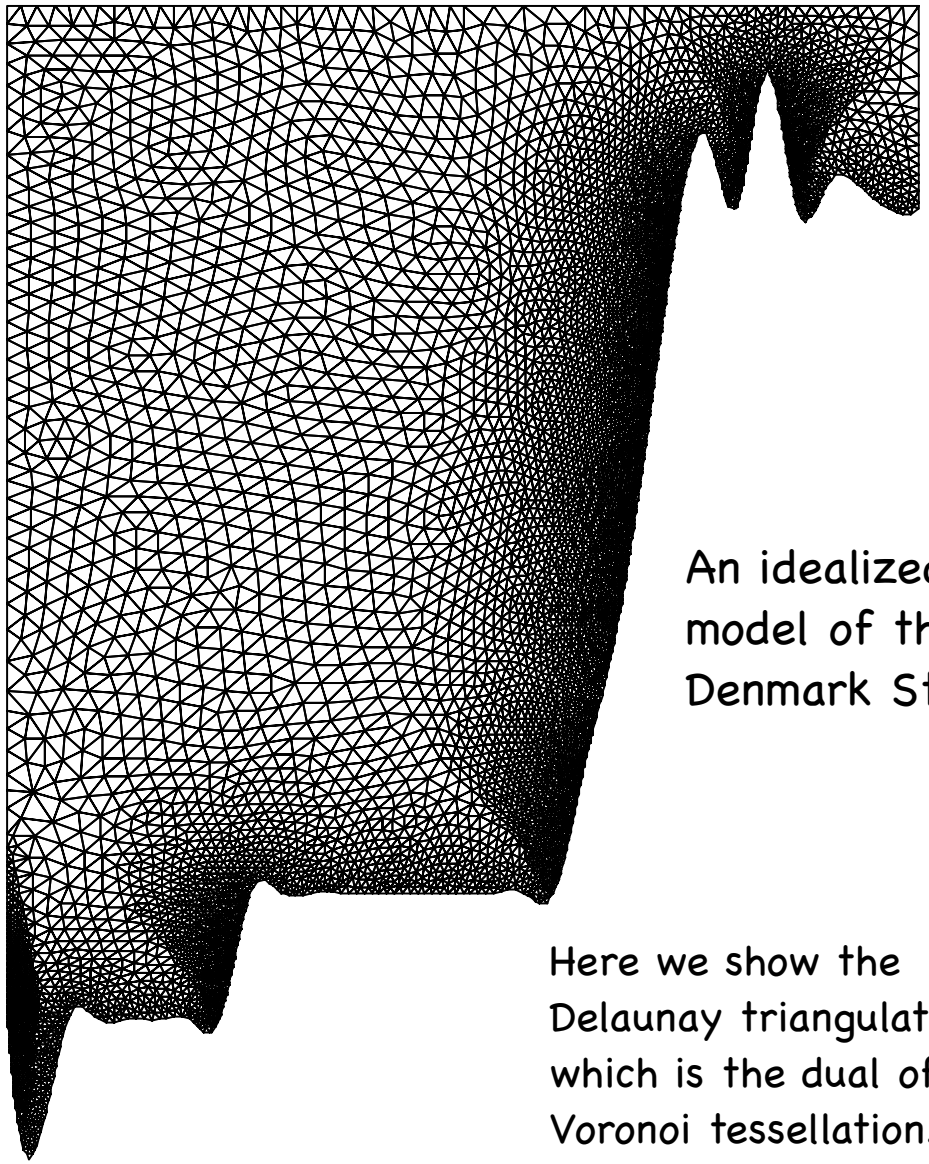
# Computational Savings

It appears that we can resolve the regions of strong eddy activity using approximately 10% of the degrees of freedom required for a global 1/10 degree simulation.

This means that IPCC-class simulations could include regional eddy activity within a decade.

This is a relatively low-risk activity.

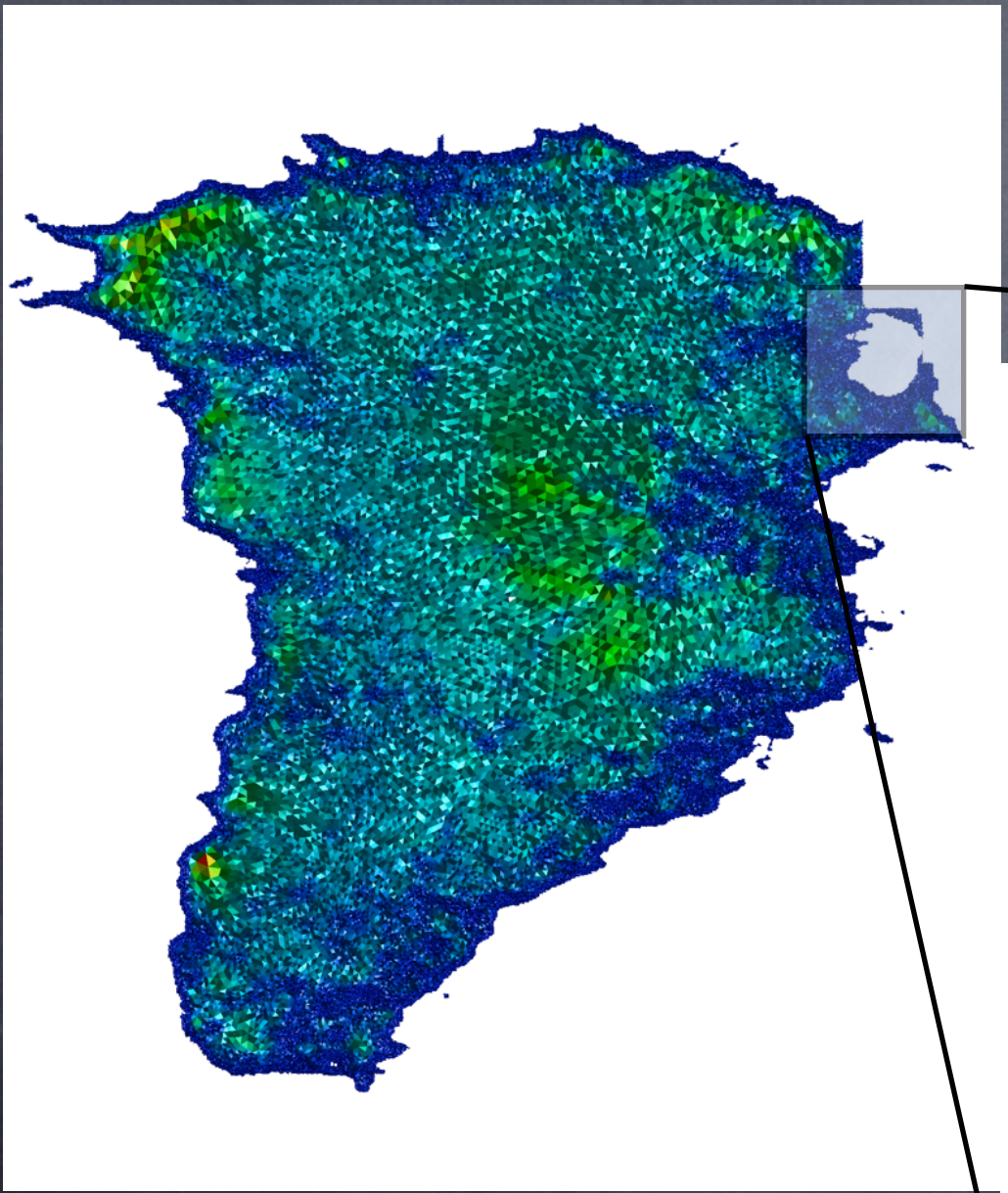
# Higher on the risk/return ladder ... unstructured 3D meshes



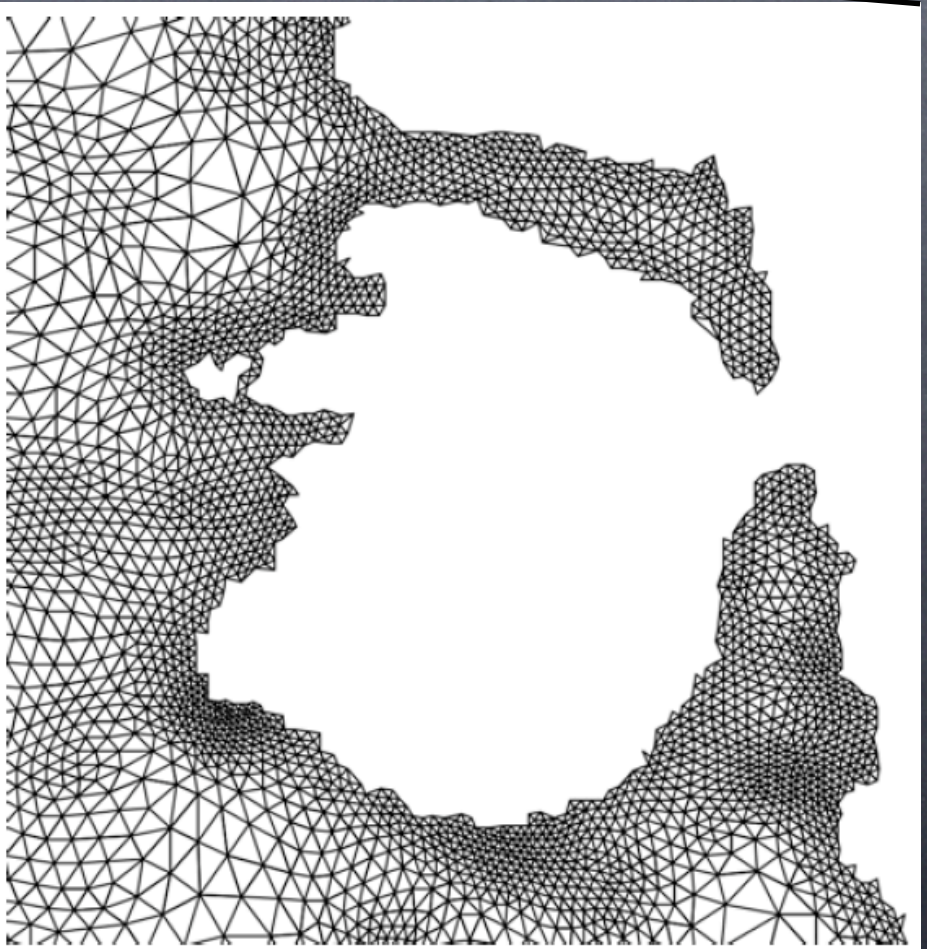
An idealized model of the Denmark Straits

Here we show the Delaunay triangulation -- which is the dual of the Voronoi tessellation.

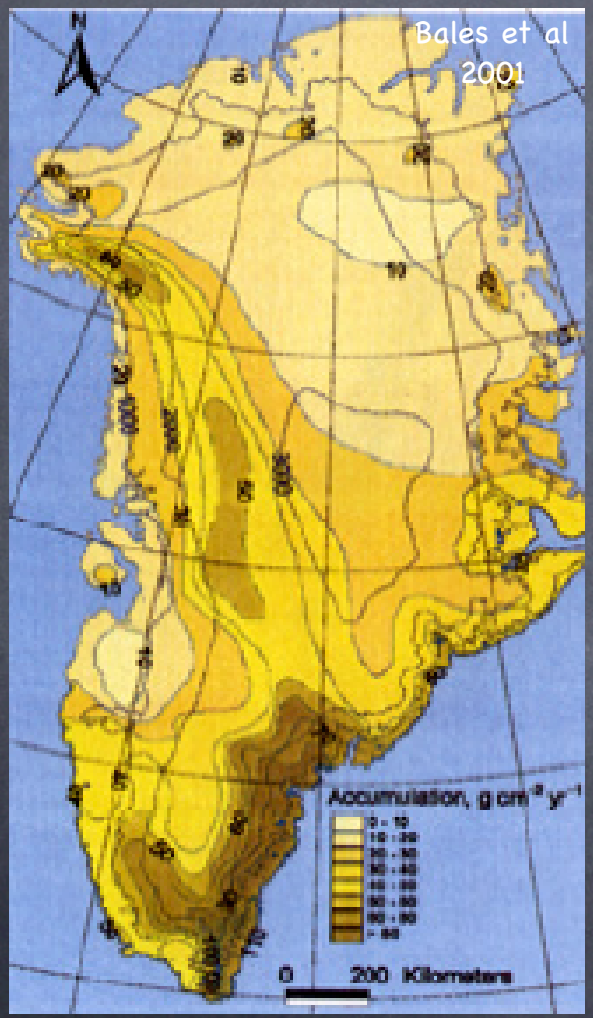
# Greenland grids …



Note: Boundary conforming



With this approach we will be able to conduct multi-century simulations with approximate 50 km resolution in the interior and 2 km resolution along ice streams and other critical regions.

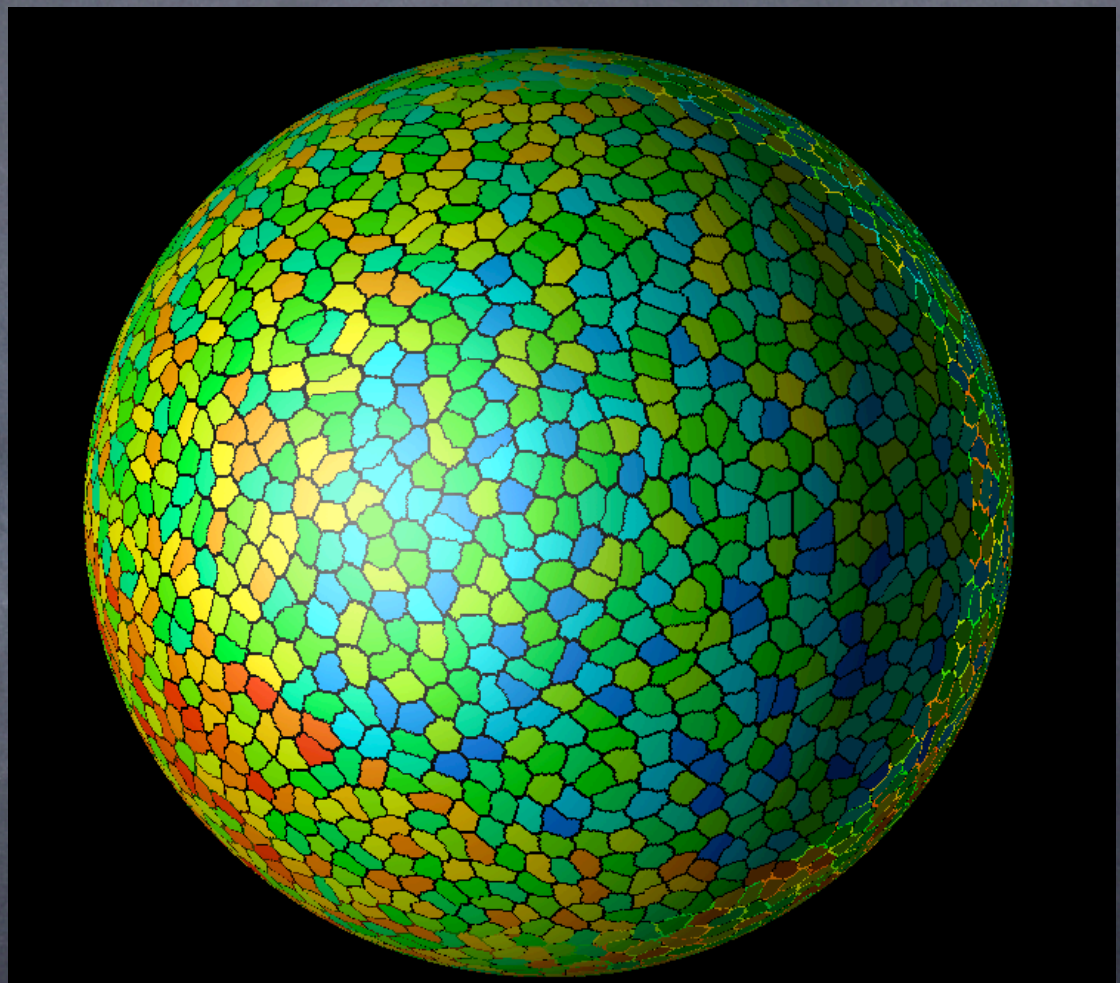


# A Design for High Performance Computing
# From the outset we are targeting efficient performance on 10,000s of processors.

Here we taken a global grid of 655362 cells (~30 km resolution) and separated it into 2562 individual blocks.

These blocks are created to balance the work-per-block and to minimize the amount of information that must be communicated between blocks.

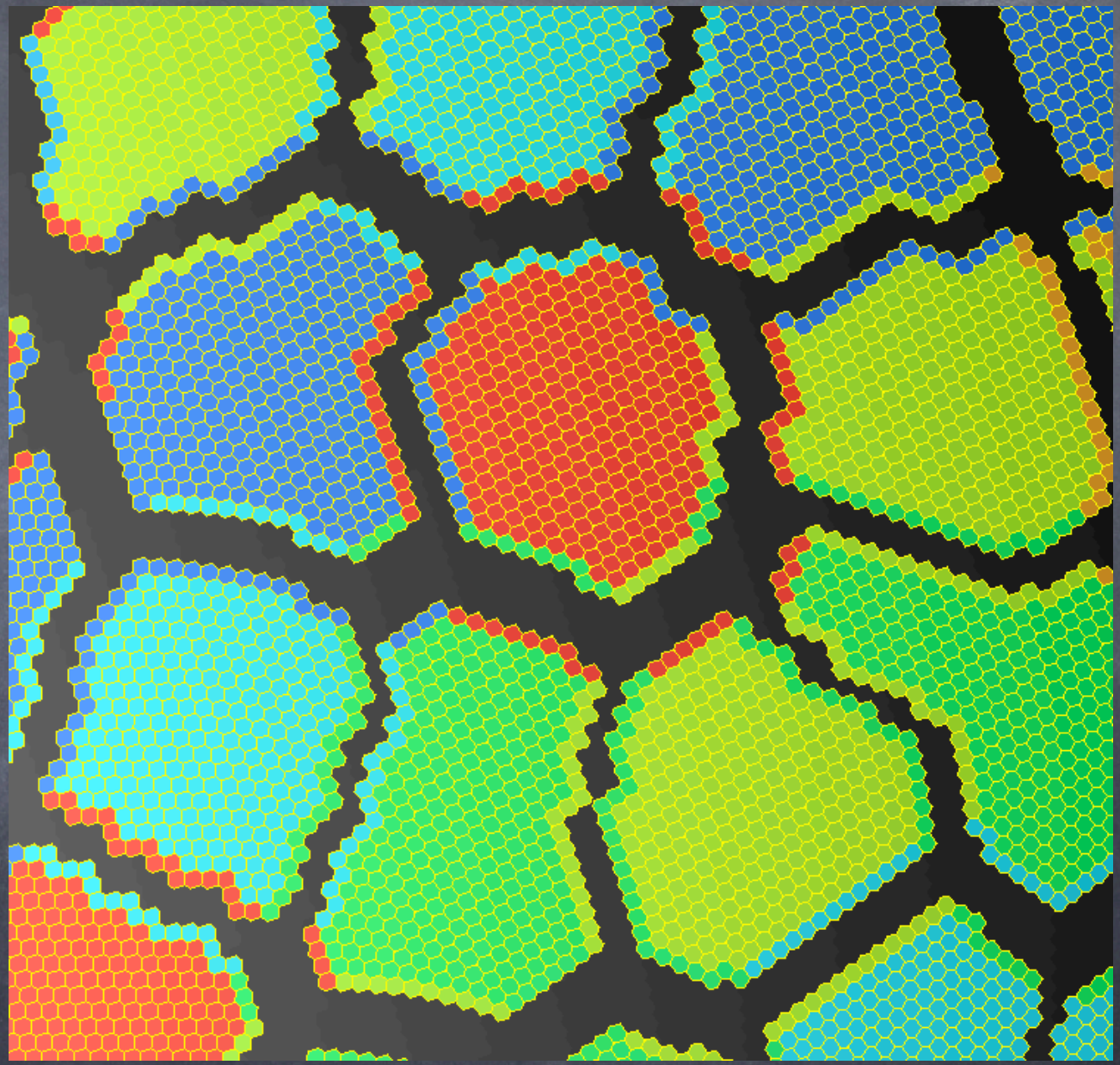
We can assign an arbitrary number of blocks per processor and, thus, support two types of parallelism within this framework (i.e. distributed memory across nodes and shared memory within a node).



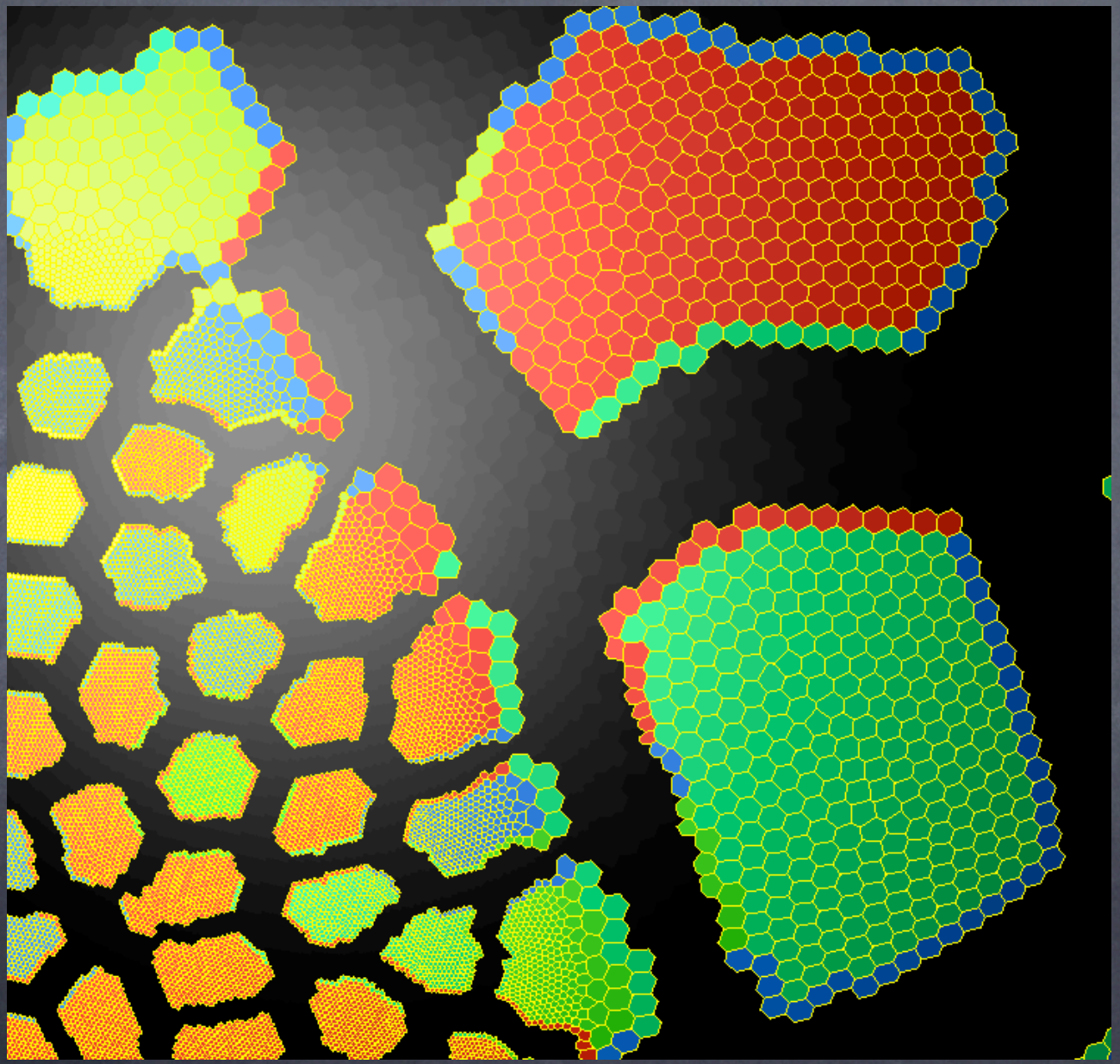
A global 30 km broken into 2562 blocks

# A Design for High Performance Computing

Close-up of block decomposition showing "ghost" cell data that indicates interblock communication.



# And on a nonuniform SCVT ....

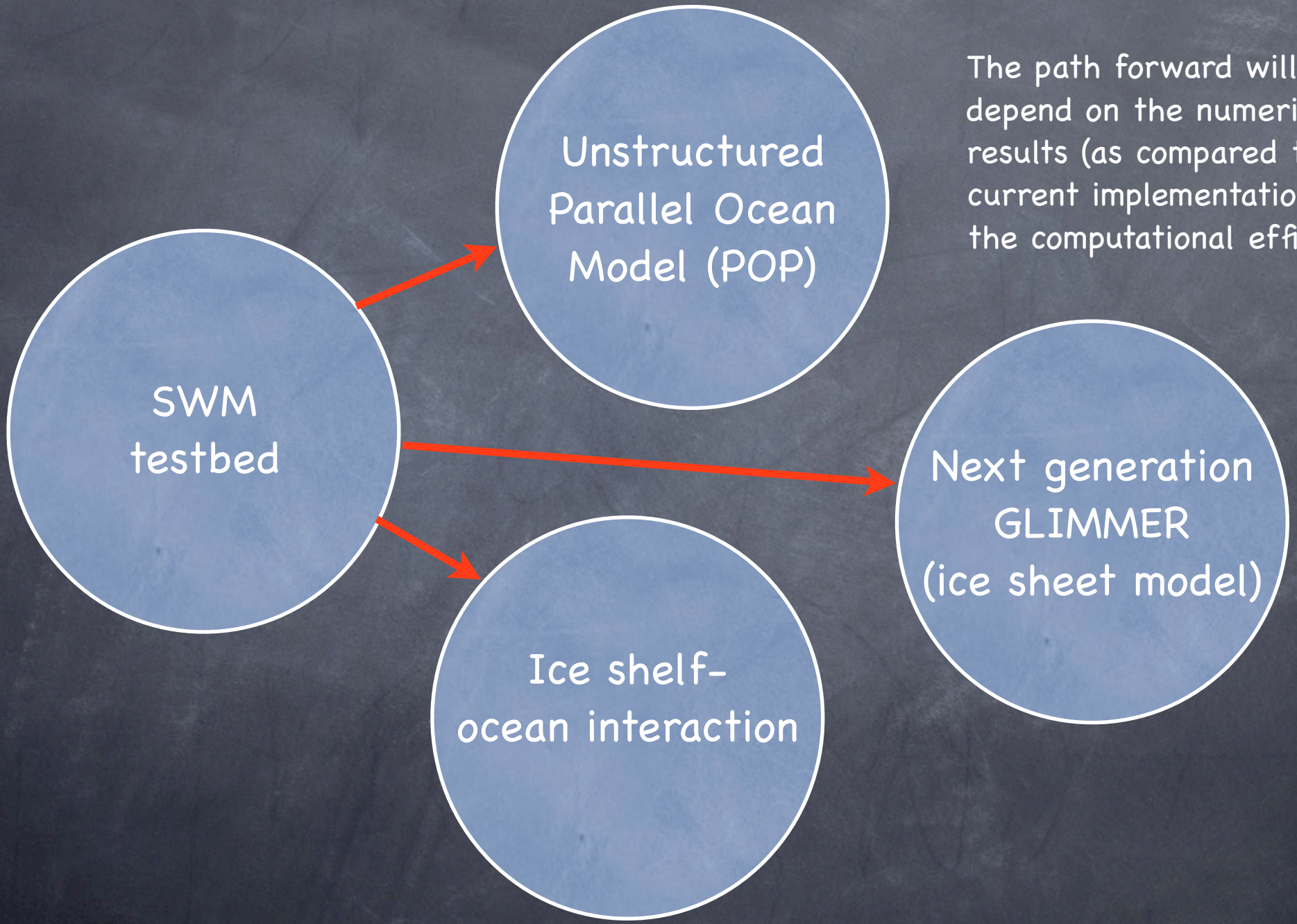


# Implementation

We are currently in a scoping process to identify a set of numerical methods that are applicable to the wide class of phenomena that were identified at the beginning of this presentation.

We have implemented this framework as a stacked shallow-water model in order to test various finite-volume and spectral-element formulations.

# Potential Development Path




The path forward will depend on the numerical results (as compared to the current implementation) and the computational efficiency.

# Summary

- Quantifying regional climate change will become a primary research activity in the next decade.
  - for assessing global climate change
  - for assessing regional impacts
- The resolution required for this increased level of fidelity will not be possible using traditional modeling approaches.
- A suite of numerical methods will be used to bridge the global to regional scales.
- We are pursuing one approach, based on variable resolution Spherical Centroidal Voronoi Tessellations.
- Preliminary results are encouraging.

Thank you.